Centro Brasileiro de Pesquisas Físicas

Cosmologia com python

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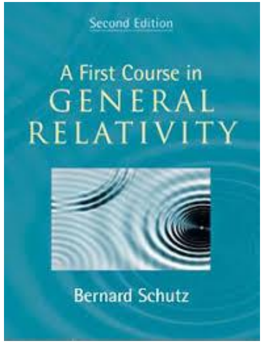
clearnightsrthebest.com

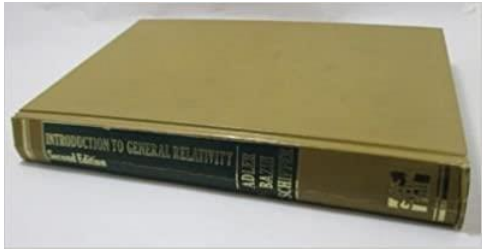
Métrica FRLW

$$ds^2 = c^2dt^2 - (dx^2 + dy^2 + dz^2)$$

Homogeneidade e isotropia....

$$ds^2 = c^2dt^2 - R(t)^2\left[\frac{d\sigma^2}{1-k\sigma^2} + \sigma^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$



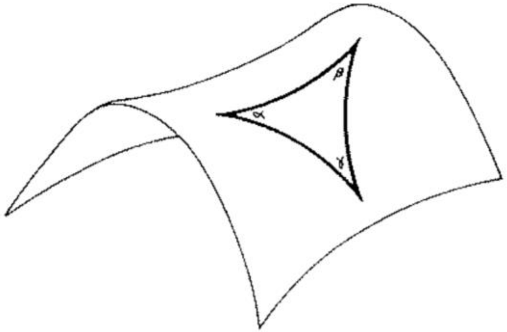


Adler Bazin

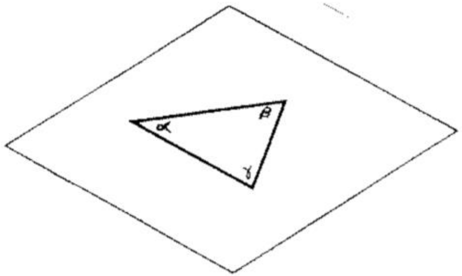
Métrica FRLW

$$ds^2 = c^2dt^2 - R(t)^2\left[\frac{d\sigma^2}{1-k\sigma^2} + \sigma^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

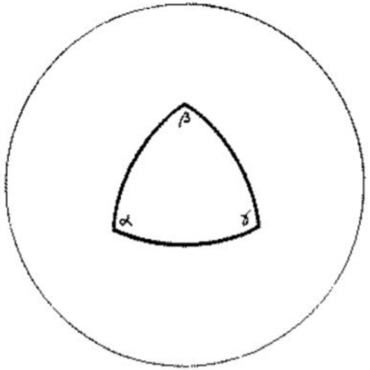
K=-1



K=0



K=1



Métrica FRLW

$$ds^2 = c^2 dt^2 - R(t)^2 \left[\frac{d\sigma^2}{1 - k\sigma^2} + \sigma^2 (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

$$ds^2 = 0$$

$$(0,0,0) \qquad\qquad (\sigma_G, 0, 0)$$

$$c\frac{dt}{R(t)} = \frac{d\sigma}{\sqrt{1 - k\sigma^2}}$$

Métrica FRLW

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$$ds^2 = 0$$

$$(0,0,0) \qquad\qquad (\sigma_G, 0, 0)$$

$$\int_0^{\sigma_G} \frac{d\sigma}{\sqrt{1 - k\sigma^2}} = c \int_{t+\Delta t}^{t_0+\Delta t_0} \frac{dt}{R(t)}.$$

$$\int_t^{t_0} \frac{dt}{R(t)} = \int_{t+\Delta t}^{t_0+\Delta t_0} \frac{dt}{R(t)}$$

Métrica FRLW

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Métrica FRLW

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$\lambda_e = c\Delta t \text{ e } \lambda_0 = c\Delta t_0$

$$\frac{\lambda_0}{\lambda_e} = \frac{R(t_0)}{R(t)} = \frac{R_0}{R}$$

Métrica FRLW

$$\frac{\Delta t}{R(t)} \simeq \frac{\Delta t_0}{R(t_0)}$$

$$\frac{\lambda_0}{\lambda_e} = \frac{R(t_0)}{R(t)} = \frac{R_0}{R}$$

$\lambda_e = c\Delta t$ e $\lambda_0 = c\Delta t_0$

$$z \equiv \frac{\lambda_0 - \lambda_e}{\lambda_e}$$

$$a(z) = \frac{R}{R_0} = \frac{1}{1+z}$$

Friedmann Equations again...

$$a(z) = \frac{R}{R_0} = \frac{1}{1+z}$$

$$= \frac{8\pi G}{3}\left(\rho_{m,0}\left(\frac{a_0}{a}\right)^3 + \rho_{r,0}\left(\frac{a_0}{a}\right)^4 + \rho_X\left(\frac{a_0}{a}\right)^{3(1+w_X)}\right) + H_0^2(1-\Omega_0)\frac{a_0^2}{a^2}$$

$$= H_0^2\left(\Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + \Omega_{X,0}(1+z)^{3(1+w_X)} + (1-\Omega_0)(1+z)^2\right)$$

Máxima Verossilhança

$$H^2 = H_0^2\left(\Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + \Omega_{X,0}(1+z)^{3(1+w_X)} + (1-\Omega_0)(1+z)^2\right)$$

A PDF correspondente para uma amostra de n variáveis aleatórias independentes e identicamente distribuídas normais (a probabilidade) é

$$f(x_1,\ldots,x_n \mid \mu,\sigma^2) = \prod_{i=1}^{n} f(x_i \mid \mu,\sigma^2) = \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} \exp\left(-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}\right),$$

$$F = \frac{L}{4\pi a^2(t_0)r_1^2(1+z)^2}$$

$$\boxed{z := \frac{\lambda_0-\lambda_1}{\lambda_1} = \frac{a(t_0)}{a(t_1)} - 1}$$

Máxima Verossilhança

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Dado um Fluxo e uma Luminosidade -→ temos uma distancia Luminosidade e um modulo de distância (Observado)

$$\boxed{d_L = a(t_0) r_1 (1+z)}$$

$$\mu = m - M = 5\log_{10}(d)$$

Dado um redshift e os parâmetros Cosmológicos, temos a distancia luminosidade do modelo

Friedmann Equations from GR

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

$$R_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\mu\alpha,\nu} + \Gamma^{\beta}_{\mu\nu}\Gamma^{\alpha}_{\beta\alpha} - \Gamma^{\beta}_{\mu\alpha}\Gamma^{\alpha}_{\beta\nu}$$

$$\Gamma^{\rho}_{\alpha\beta} = \frac{1}{2}g^{\rho\lambda}(g_{\alpha\lambda,\beta} + g_{\lambda\beta,\alpha} - g_{\beta\alpha,\lambda})$$

$$\begin{aligned}
R_{00} &= \cancel{\Gamma^{\alpha}_{00,\alpha}} - \Gamma^{\alpha}_{0\alpha,0} + \cancel{\Gamma^{\beta}_{00}\Gamma^{\alpha}_{\beta\alpha}} - \Gamma^{\beta}_{0\alpha}\Gamma^{\alpha}_{\beta 0} \\
&= -\Gamma^{\alpha}_{0\alpha,0} - \Gamma^{\beta}_{0\alpha}\Gamma^{\alpha}_{\beta 0} \\
&= -\Gamma^{i}_{0i,0} - \Gamma^{j}_{0i}\Gamma j0^{i} \\
&= -\frac{\partial}{\partial t}\left[\frac{\dot{a}}{a}\delta^i_i\right] - \left[\left(\frac{\dot{a}}{a}\right)^2 \delta^j_i \delta^i_j\right] \\
&= -3\frac{\ddot{a}}{a}
\end{aligned}$$

Friedmann Equations from GR

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&= -\Gamma^{\alpha}_{0\alpha,0} - \Gamma^{\beta}_{0\alpha}\Gamma^{\alpha}_{\beta 0} \\
&= -\Gamma^{i}_{0i,0} - \Gamma^{j}_{0i}\Gamma j0^{i} \\
&= -\frac{\partial}{\partial t}\left[\frac{\dot{a}}{a}\delta^{i}_{i}\right] - \left[\left(\frac{\dot{a}}{a}\right)^{2}\delta^{j}_{i}\delta^{i}_{j}\right] \\
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$$\Gamma^{\rho}_{\alpha\beta} = \frac{1}{2} g^{\rho\lambda}\left(g_{\alpha\lambda,\beta} + g_{\lambda\beta,\alpha} - g_{\beta\alpha,\lambda}\right)$$

$$\begin{aligned}
R_{0i} = R_{i0} &= \Gamma^{\alpha}_{i0,\alpha} - \Gamma^{\alpha}_{i\alpha,0} + \Gamma^{\beta}_{i0}\Gamma^{\alpha}_{\beta\alpha} - \Gamma^{\beta}_{i\alpha}\Gamma^{\alpha}_{\beta j} \\
&= \Gamma^{\alpha}_{i0,\alpha} - \Gamma^{j}_{ij,0} + \Gamma^{j}_{i0}\Gamma^{k}_{jk} - \Gamma^{j}_{ik}\Gamma^{k}_{j0} \\
&= \cancel{\frac{\partial}{\partial x^j}\left(\frac{\dot a}{a}\right)\delta^j_i} - \cancel{\frac{\partial}{\partial t}\Gamma^j_{ij}} + \left(\frac{\dot a}{a}\right)\left[\delta^j_i \Gamma^k_{jk} - \delta^k_j \Gamma^j_{ik}\right] \\
&= \left(\frac{\dot a}{a}\right)\left[\Gamma^k_{ik} - \Gamma^k_{ik}\right] \\
&= 0
\end{aligned}$$

Friedmann Equations from GR

$$R_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\mu\alpha,\nu} + \Gamma^{\beta}_{\mu\nu}\Gamma^{\alpha}_{\beta\alpha} - \Gamma^{\beta}_{\mu\alpha}\Gamma^{\alpha}_{\beta\nu}$$

$$\Gamma^{\rho}_{\alpha\beta} = \frac{1}{2} g^{\rho\lambda}\left(g_{\alpha\lambda,\beta} + g_{\lambda\beta,\alpha} - g_{\beta\alpha,\lambda}\right)$$

$$R_{ii} = \left(\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + \frac{2k}{a^2}\right) g_{ii}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\begin{aligned} R &= g^{\mu\nu} R_{\mu\nu} \\ &= -R_{00} + g^{11} R_{11} + g^{22} R_{22} + g^{33} R_{33} \\ &= 3\frac{\ddot{a}}{a} + 3\left(\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + \frac{2k}{a^2}\right) \\ &= 6\frac{\ddot{a}}{a} + 6\left(\frac{\dot{a}}{a}\right)^2 + 6\frac{k}{a^2} \end{aligned}$$

Friedmann Equations from GR

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R.$$

$$\boxed{\begin{aligned} G_{00} &= R_{00} - \frac{1}{2} R g_{00} \\ &= 3\left(\frac{\dot{a}}{a}\right)^2 + \frac{3k}{a^2} \end{aligned}}$$

$$\boxed{\begin{aligned} G_{ii} &= R_{ii} - \frac{1}{2} R g_{ii} \\ &= -\left(2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2}\right) g_{ii} \end{aligned}}$$

Friedmann Equations from GR

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$c^{-2} \cdot$ (energy density)

momentum density

$$\begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

shear stress

pressure

energy flux

momentum flux

$$T^{\mu\nu} = (\rho c^2 + p) u^\mu u^\nu + p \eta^{\mu\nu}$$

$$T^{\mu\nu}_{;\nu} = 0$$

Friedmann Equations from GR

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Fluído perfeito, homogeneidade e isotropia

$$T^{\mu\nu} = \begin{pmatrix} \rho(t)c^2 & 0 & 0 & 0 \\ 0 & p(t) & 0 & 0 \\ 0 & 0 & p(t) & 0 \\ 0 & 0 & 0 & p(t) \end{pmatrix}$$

$$T^{\mu\nu} = (\rho c^2 + p)u^\mu u^\nu + p\eta^{\mu\nu}$$

$$T^{\mu\nu}_{;\nu} = 0$$

Friedmann Equations from GR

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Fluído perfeito, homogeneidade e isotropia

$$T^{\alpha\beta} = \left(\rho + \frac{p}{c^2}\right)u^\alpha u^\beta + pg^{\alpha\beta}$$

$$T^{\mu\nu}_{;\nu} = 0$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p/c^2) = 0$$

Friedmann Equations from GR

$$T^{\mu\nu}_{;\nu} = 0$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p/c^2) = 0$$

Exercício mostrar que:

- matéria ("poeira", $p = 0$): $\rho_m \propto a^{-3}$
- radiação ($p = \rho_r c^2/3$): $\rho_r \propto a^{-4}$
- vácuo: $p = -\rho_v c^2$, $\rho_v = \Lambda/(8\pi G)$
- modelo simples para energia escura: $p = w\rho c^2$, com w constante

Friedmann Equations from GR

$$p = w\varepsilon = w\rho c^2$$

$$\boxed{\rho \propto a^{-3(1+w)}}$$

$$\rho \propto \begin{cases} a^{-3} & \text{non-relativistic matter}(w=0) \\ a^{-4} & \text{radiation}(w=1/3) \\ a^{0} & \text{cosmological constant}(w=-1) \end{cases}$$

Ministério da Ciência, Tecnologia e Inovações

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